

Exercise 7: Black Body Radiation

the `for` loop and numerical integration

Introduction to the `for` loop:

In the examples we have seen so far the commands are executed consecutively, working down the script from the top to the bottom. In real programs you may want to repeat the same command several times, or perform different calculations in different situations. Statements that tell the computer which command is to be executed next are fundamental building blocks for computer programs, and are called control-flow statements. In this exercise you will learn how to use the “`for` loop”, for repeating a statement or group of statements a fixed number of times,

A simple form of such a loop is

```
for index = 1:n
    statements
end
```

The statements are executed n times with $\text{index} = 1, 2, 3, \dots, n$. Here n is a number that must be fixed before we enter the loop. More generally, the loop may start with the line `for index = j:m:k`. Usually j, m and k are integers and $k-j$ is divisible by m . The statements are executed repeatedly starting from $\text{index} = j$ with index incremented by m each time and continuing until $\text{index} = k$ (which is the last time round the loop). If $m = 1$, we can replace `j:m:k` in the first line of the loop by `j:k`. Even more generally, we can begin with the line `for index = v`, where v is a vector. The statements inside the loop are repeatedly executed with index set to each of the elements of v in turn. Type `help for` and read the manual page for the `for` loop.

Exercise

The radiation intensity per unit area of a black body radiator in the wavelength range λ to $\lambda + d\lambda$ is $I(\lambda)d\lambda$ and is governed by Planck's radiation formula

$$I(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \left(\frac{1}{e^{\frac{hc}{\lambda k T}} - 1} \right)$$

where

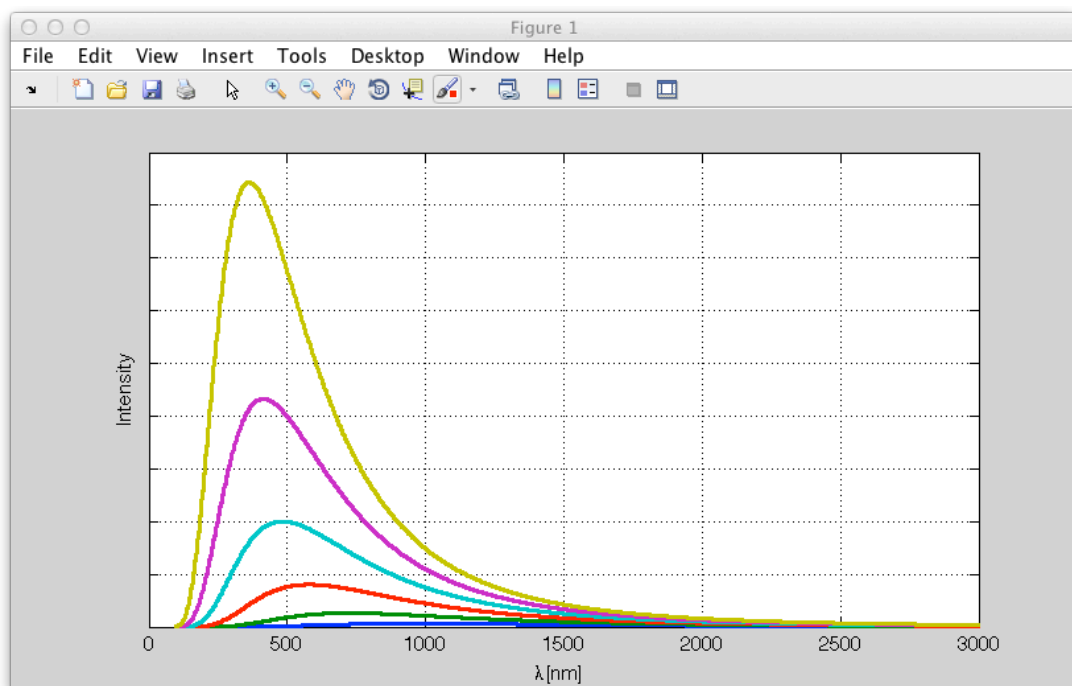
h is Planck's constant	6.6260 10 ⁻³⁴ J s
k is Boltzmann's constant	1.3806 10 ⁻²³ J/K
c is the velocity of light	2.9979 10 ⁸ m/s
T is the absolute temperature	K

A. Construct a plot showing the Black Body spectra at a range of temperatures. Use a function to compute the 2-dimensional array $I(\lambda, T)$. A suitable temperature range is 3000 K to 8000 K in steps of, say, 1000 K with a wavelength range of 100 nm to 3000 nm divided into 1000 intervals.

Define an array with temperatures T , and call the function that computes the intensity inside a `for` loop.

You can then plot I for each value j of temperature inside the `for` loop (e.g. `plot(λ , I(:, j))`) or use a single plot command outside the loop (e.g. `plot(λ , I)`).

Plotting I against λ should result in a chart similar to the following:



B. Use the MATLAB function `max` to compute the wavelength at the maximum intensity for each temperature. Compare the wavelength of visible light with that corresponding to the maximum radiation intensity of the Sun (whose radiation spectrum can be approximated to that of a black body of $T \approx 6000$ K).

Tip: The function `max(A)` gives as output not only the maximum value of A , but also the index of the maximum values of A .

C. Read the online documentation on how to use the numerical integration routine `trapz`, and use it to estimate the area under each of the curves i.e. the total intensity.

Stefan's Constant (σ_T) can be calculated from the relationship $I_T = \sigma_T T^4$. The accepted value of Stefan's constant (σ) is $5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$. Compare this value with your calculations for various temperatures and wavelength ranges. To improve the value you obtain for σ_T try increasing the upper limit of the wavelength range considered.

When the exercise is complete save your work and show it to a demonstrator.

Further reading:

see Chapter 2 (in particular 2.7) in the e-book “Essential Matlab for Engineers and Scientists”

<http://www.sciencedirect.com/science/article/pii/B9780123748836000023>